Note Title 09/10/2007

C1 Exercise 2D (solve quadratics with completing square)

1 $x^2 + 6x + 1 = 0$

$(x+3)^2 - 9 + 1 = 0$

$(x+3)^2 - 8 = 0$

$(x+3)^2 = 8$

$x + 3 = \pm\sqrt{8}$

$x = -3 \pm \sqrt{8}$

if I do $(x+3)(x+3)$ I get $x^2 + 3x + 3x + 9$ $= x^2 + 6x + 9$ and I don't want the 9.

now simplify the surd since $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

(if that's gone over your head return to C1 Ex 1G and 1H.

so $x = -3 \pm 2\sqrt{2}$

2 $x^2 + 12x + 3 = 0$

$(x+6)^2 - 36 + 3 = 0$

$(x+6)^2 - 33 = 0$

$(x+6)^2 = 33$

$x + 6 = \pm\sqrt{33}$

$x = -6 \pm \sqrt{33}$

3 $x^2 - 10x = 5$

$$x^2 - 10x - 5 = 0$$

$$(x - 5)^2 - 25 - 5 = 0$$

$$(x - 5)^2 - 30 = 0$$

$$(x - 5)^2 = 30$$

$$x - 5 = \pm\sqrt{30}$$

$$x = 5 \pm \sqrt{30}$$

4 $x^2 + 4x - 2 = 0$

$$(x+2)^2 - 4 - 2 = 0$$

$$(x+2)^2 - 6 = 0$$

$$(x+2)^2 = 6$$

$$(x+2) = \pm\sqrt{6}$$

$$x = -2 \pm \sqrt{6}$$

5 $x^2 - 3x - 5 = 0$

$\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - 5 = 0$

$\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{20}{4} = 0$

$\left(x - \frac{3}{2}\right)^2 - \frac{29}{4} = 0$

$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$

$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{\sqrt{4}} = \frac{\pm\sqrt{29}}{2}$

so $x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$ or $x = \frac{3 \pm \sqrt{29}}{2}$ if you prefer.

if I do $\left(x - \frac{3}{2}\right)\left(x - \frac{3}{2}\right)$

I get $x^2 - \frac{3}{2}x - \frac{3}{2} + \frac{9}{4}$

and so $x^2 - \frac{3\cancel{6}}{\cancel{2}}x + \frac{9}{4}$

6 $2x^2 - 7 = 4x$

$$2x^2 - 4x - 7 = 0$$

$$x^2 - 2x - \frac{7}{2} = 0$$

$$(x-1)^2 - 1 - \frac{7}{2} = 0$$

$$(x-1)^2 - \frac{2}{2} - \frac{7}{2} = 0$$

$$(x-1)^2 - \frac{9}{2} = 0$$

$$(x-1)^2 = \frac{9}{2}$$

So $x - 1 = \pm\sqrt{\frac{9}{2}} = \frac{\pm\sqrt{9}}{\sqrt{2}} = \frac{\pm 3}{\sqrt{2}}$

since $x - 1 = \frac{\pm 3}{\sqrt{2}}$

$$x = 1 \pm \frac{3}{\sqrt{2}}$$

but you can't leave it here
you must simplify the surd by
"rationalising the denominator"
see C1 Ex 1H.

$$x = \frac{\sqrt{2} \pm 3}{\sqrt{2}} \times \underbrace{\frac{\sqrt{2}}{\sqrt{2}}}_{=1}$$

$$x = \frac{2 \pm 3\sqrt{2}}{2}$$

or $x = 1 \pm \frac{3}{2}\sqrt{2}$

7 $4x^2 - x = 8$

$$4x^2 - x - 8 = 0$$

$$x^2 - \frac{1}{4}x - 2 = 0$$

$$\left(x - \frac{1}{8}\right)^2 - \frac{1}{64} - \frac{128}{64} = 0$$

$= 2$ so that denominators match.

$$\left(x - \frac{1}{8}\right)^2 = \frac{129}{64}$$

$$x - \frac{1}{8} = \frac{\pm\sqrt{129}}{8}$$

$$x = \frac{1 \pm \sqrt{129}}{8}$$

8 $10 = 3x - x^2$

$$x^2 - 3x + 10 = 0$$

$$\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{40}{4} = 0$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{-31}{4}$$

$$x - \frac{3}{2} = \frac{\pm\sqrt{-31}}{2}$$

$$x = \frac{3 \pm \sqrt{-31}}{2} \; *$$

since the value inside the root is negative we conclude the equation has no <u>real</u> roots.

* due to a misprint in early editions C1 Ex 1E Q8 led to us discussing this issue already. See p25 onward in the solutions to this exercise.
An Autograph file created for that question shows the existence of roots in the complex x-y plane with the z axis showing the modulus of $f(x,y)$ as a surface.

9 $15 - 6x - 2x^2 = 0$

$$2x^2 + 6x - 15 = 0$$

$$x^2 + 3x - \frac{15}{2} = 0$$

$$\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{30}{4} = 0$$

$$\left(x + \frac{3}{2}\right)^2 - \frac{39}{4} = 0$$

$$x + \frac{3}{2} = \pm\frac{\sqrt{39}}{2}$$

$$x = \frac{-3 \pm \sqrt{39}}{2}$$

10 $5x^2 + 8x - 2 = 0$

$$x^2 + \frac{8}{5}x - \frac{2}{5} = 0$$

$$\left(x + \frac{4}{5}\right)^2 - \frac{16}{25} - \frac{10}{25} = 0$$

$$\left(x + \frac{4}{5}\right)^2 - \frac{26}{25} = 0$$

$$x + \frac{4}{5} = \pm \frac{\sqrt{26}}{5}$$

$$x = \frac{-4 \pm \sqrt{26}}{5}$$